Chapter 20 Induced Voltages and Inductance

§20.1 Induced emf and Magnetic Flux
感應 電動勢　　磁通量

§. Magnetic Flux 磁通量

magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta$

其中 $\vec{B}$：磁場強度（或稱磁通密度）

$d\vec{A}$：面積向量（即垂直平面的法線）（面積大小 $= dA$）

Example 20.1

§20.2 Faraday's Law of Induction

Faraday's Law of induction, can be written

$$\left(\begin{array}{c}\text{induced}\\ \text{emf}\end{array}\right) \quad \varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} \quad (N\text{：線圈的匝數（圈數）})$$

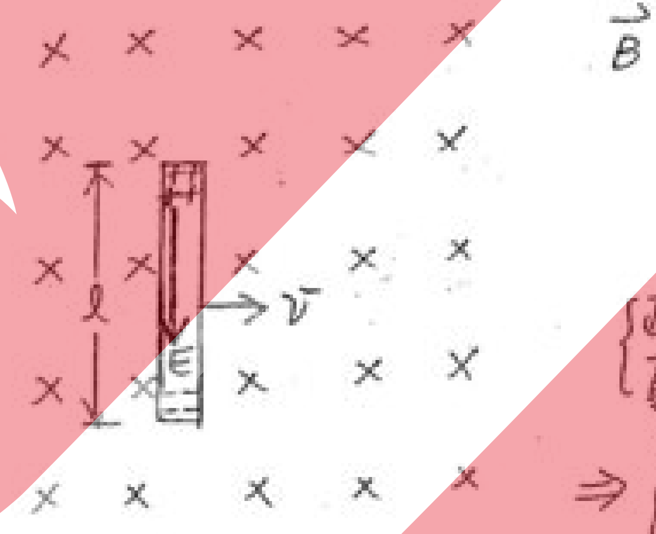
（感應電動勢）

Example 20.2

§20.3 Motional emf

Fig 20.10（電腦教材）

考慮在均勻磁場中運動的導體



導體中的電荷受磁力 $= qvB$

$\Big\{$正電荷所受磁力方向為向上；負電荷所受磁力方向為向下

$\Rightarrow \Big\{$正電荷往導體上端堆積；負電荷往導體下端堆積.

$\Rightarrow$ 形成導體內部有電場存在，而電場方向為正電荷指向負電荷，

即 $\downarrow E$.

達平衡時：電場的電力 $=$ 磁力

即 $qE = qvB$　　$\therefore E = vB$

而感應電動勢 $\varepsilon = El$　　（l：導體長度）

$\therefore \varepsilon = vBl$

若將導體接上線圈，假設線圈電阻為 R

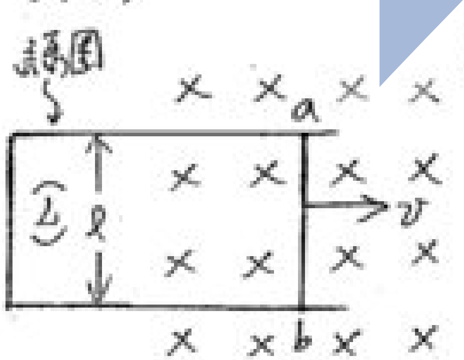
則感應電流 $I = \frac{\varepsilon}{R} = \frac{vBl}{R}$

功率 $P = (IlB)v = \frac{B^2l^2v^2}{R} = \left(\frac{Blv}{R}\right)^2 R = I^2R$

Example 20.3

南台科技大學
Southern Taiwan University

§20.4 Lenz's Law Revisited (The Minus Sign in Faraday's Law)

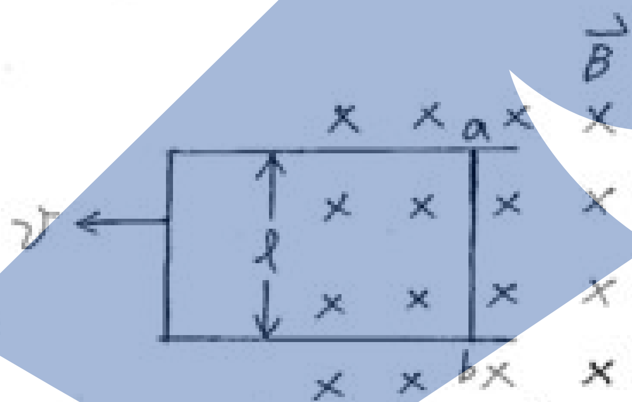


試判斷出感應電流的方向:

① 後來的磁通量 > 原來的磁通量

② 根據冷次定律(或 Lenz's Law)判斷出應生的感應磁場方向必須為 出紙面

③ 根據右手定則可判斷出感應電流方向必須為 逆時針方向 (b→a)



試判斷出感應電流的方向:

① 後來的磁通量 < 原來的磁通量

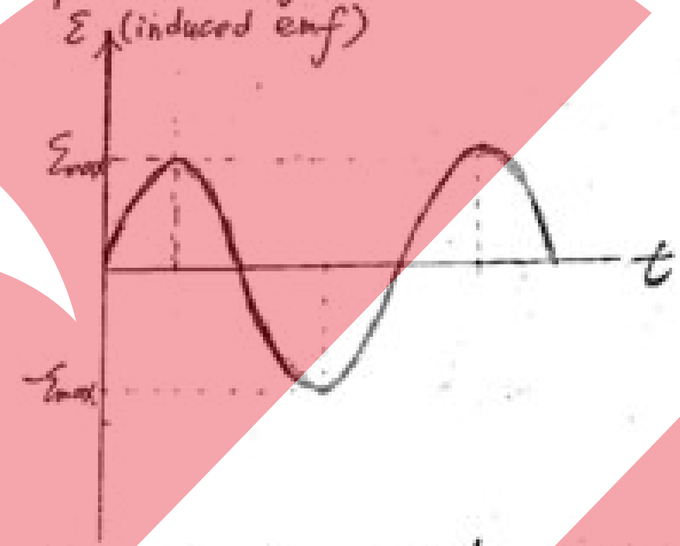
② 根據冷次定律(或 Lenz's Law)判斷出應生的感應磁場方向必須為 入紙面

③ 根據右手定則可判斷出感應電流方向必須為 順時針方向 (a→b)

§20.5 Generators (發電機)

(一) The alternating current (AC) generators (交流發電機)

(p.531) Fig. 20.15



$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d(\cos\omega t)}{dt} = NBA\omega\sin\omega t$$

$\varepsilon_{max.} = NBA\omega$ (when $\omega t = 90°$ or $270°$)

(二) The direct current (DC) generator (直流發電機)

(p.532) Fig 20.17

Example 20.4

南台科技大學 Southern Taiwan University

§20.6 Self-inductance 自感

The self-induced emf is always proportional to the time rate of change of the current.

($\mathcal{E} \propto \frac{dI}{dt}$)

(p.535) Fig 20.19

線圈自己通過電流發生改變 ⇒ 線圈內磁場變化

⇒ 線圈自己產生感應電動勢(感應電流)

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

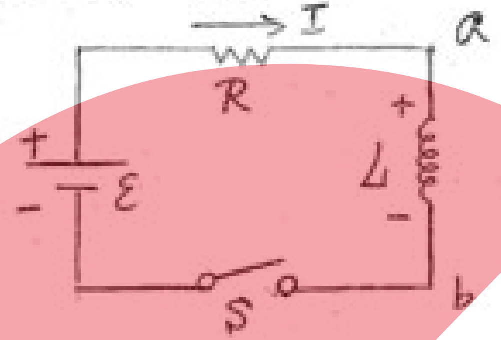
L: 自感係數 (單位為亨利 henry (H))

$$L = N\frac{\Delta\Phi_B}{\Delta I} = N\frac{\Phi_B}{I}$$

$$\left(L = -\frac{\mathcal{E}}{(dI/dt)}\right)$$

Example 20.6 (參考 §19.8)

§20.7 RL Circuits



(感應) $\mathcal{E}_L = -L\frac{dI}{dt}$

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$\frac{\mathcal{E}}{R} - I - \frac{L}{R}\frac{dI}{dt} = 0$$

令 $x = \left(\frac{\mathcal{E}}{R}\right) - I$ 則 $dx = -dI$

$\therefore x + \frac{L}{R}\frac{dx}{dt} = 0 \Rightarrow \frac{L}{R}\frac{dx}{dt} = -x \Rightarrow$

$\frac{dx}{dt} = -\frac{R}{L}x \Rightarrow \frac{dx}{x} = -\frac{R}{L}dt \Rightarrow$

$\int_{x_i}^{x}\frac{dx}{x} = -\frac{R}{L}\int_0^t dt \Rightarrow \ln\frac{x}{x_i} = -\frac{R}{L}t$

at $t=0$ $x_i = \frac{\mathcal{E}}{R}$, $I=0$

$$x = x_i e^{-\frac{Rt}{L}}$$

即 $\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R}e^{-\frac{Rt}{L}}$ (分別將 x 與 x_i 值代入)

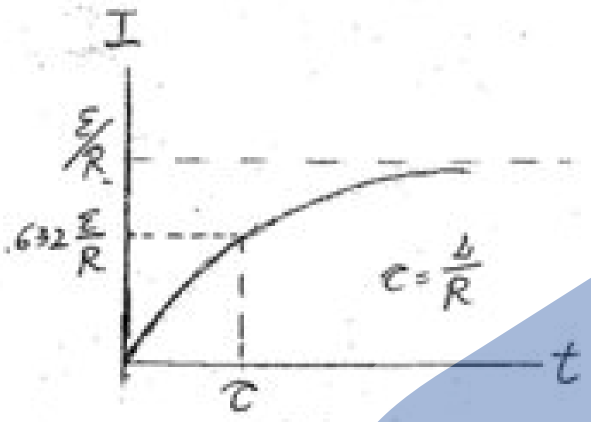
$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

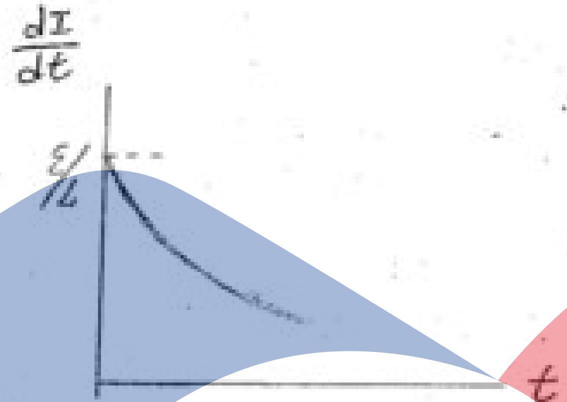
即 $I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$

其中 $\tau = \frac{L}{R}$ (τ 稱為 time constant)

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

南台科技大學
Southern Taiwan University





<u>Example 20.7</u>

(補充): Energy Stored in a Magnetic Field

in Section (16.9): we find that the energy stored by an inductor is $PE_L = \frac{1}{2}LI^2$

(in §16.9: the energy stored in a charged capacitor: $PE_C = \frac{1}{2}C(\Delta V)^2$)

南台科技大學
Southern Taiwan University