Chapter 16 Electrical Energy and Capacitance
(電 能 電容)

§16.1 Potential Difference and Electric Potential
(電位 差 電 位)

(補充): potential energy

(1) 設兩點電荷 q, q' 相距無窮遠時, 電位能 $U=0$

則兩點電荷 q, q' 相距 r 時, 電位能 $U=\frac{k_e q q'}{r}$

(代入此式時需寫上 q, q' 之正負號; $k_e \doteq 9\times10^{9}\ N\cdot m^2/C^2$)

(2) 電位能為純量, 但可能為正值或負值或零.

(3) 電位能的單位在 SI 制中為焦耳 (J), 另一常用單位為電子伏特, 簡寫為 eV. 定義: $1eV=1.6\times10^{-19}$ J.

(4) (從初位置A移至末位置B):

{ 電力作功 = 初位置電位能 U_A − 末位置電位能 U_B

{ 外力作功 = − 電力作功 = 電位能改變 $\Delta U = U_B - U_A$

即外力作功 $W_{AB} = \Delta U = \int_A^B \vec F_{外力}\cdot d\vec s$ (位移) $= -\int_A^B \vec F_{電力}\cdot d\vec s$ (位移)

$= -\int_A^B q_0 \vec E\cdot d\vec s = -q_0\int_A^B \vec E\cdot d\vec s$

電位之定義:

(1) 某一點之電位 (electric potential) V 為該點上微小測試電荷之電位能 U 除以此電荷量 q_0, 即 $V=\frac{U}{q_0}$ ($\Rightarrow U=q_0V$) Example 16.1

(2) 某點之電位 V 為將單位正電荷由無窮遠處移至該點所需的功,

即 $V \equiv \frac{功\ W}{移動電荷\ q_0}$ ($\Rightarrow W=q_0V$)

而功 $W=-\int_\infty^P q_0\vec E\cdot d\vec s$

∴ V_P (表示P點之電位) $= \frac{-\int_\infty^P q_0\vec E\cdot d\vec s}{q_0} = -\int_\infty^P \vec E\cdot d\vec s$

電位為純量, 但有可能為正值或負值或零.

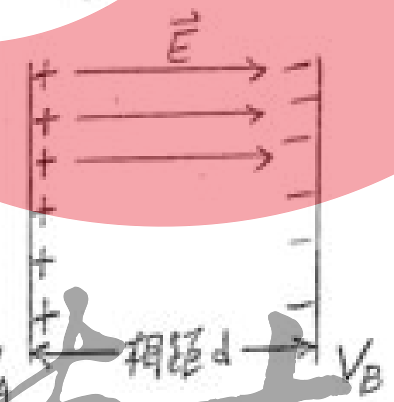
電位的單位在 SI 制中為 J/C = 伏特 (Volt)(V)

§ 電位差 (potential difference) ΔV

$\Delta V = V_B - V_A$

$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec E\cdot d\vec s$ (由電位之定義得知)

§ Potential differences in a uniform electric field



$\Delta V = V_B - V_A = -\int_A^B \vec E\cdot d\vec s$

(當 E 為定值) $= -E\int_A^B ds = -E\cdot d$

$\Delta U = -q_0\int_A^B \vec E\cdot d\vec s$

$= -q_0 E\cdot d$

$= q_0\,\Delta V$ (由補)

Example 16.3

Example 16.4

16.2 Electric Potential and Potential Energy due to Point Charges

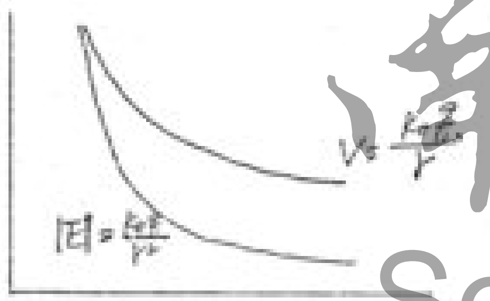
$$V = k_e \frac{q}{r}$$

$$V = k_e \sum_i \frac{q_i}{r_i} \quad (\because V 為純量)$$

$$U = \frac{k_e q q'}{r} \quad 或 \left(PE = \frac{k_e q_1 q_2}{r}\right)$$

$$U = \sum_i U_i \quad (\because U 為純量)$$

Fig. 16.5



§16.3 Potentials and Charged Conductors

外力作功 = 電位能改變 ΔPE = 末位置電位能 − 初位置電位能

$$= qV_B - qV_A = q(V_B - V_A)$$

電力作功 $= -\Delta PE = -q(V_B - V_A)$

The Electron Volt:

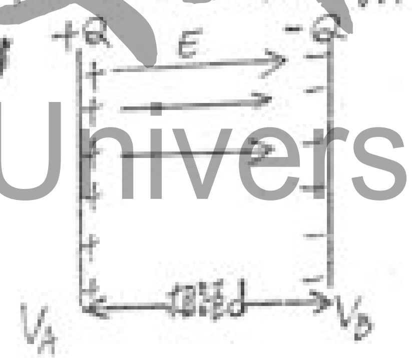
$$1\,eV = 1.6\times10^{-19}\ C\cdot V = 1.6\times10^{-19}\ J$$

§16.5 Capacitance 電容

定義：電容 $C \equiv \dfrac{每一平板上帶有電荷 Q(q)}{兩板間電位差 \Delta V}$ $[Q(q) = CV]$

電容的單位在SI制為 $\dfrac{coul\,(C)}{volt\,(V)}$ = 法拉 (F)

Fig 16.11



設每一平板面積為A

由高斯定律 $\oint \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}\sum q_i$

$$\therefore E\cdot A = \frac{Q}{\epsilon_0}$$

$$(\epsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2)$$

$\therefore E = \frac{Q}{\epsilon_0 A}$

兩板間電位差 $\Delta V = E \cdot d = \frac{Qd}{\epsilon_0 A}$

電容 $C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$

§16.6 The Parallel-Plate Capacitor 平行板電容器

$$C = \epsilon_0 \frac{A}{d}$$

Example 16.5

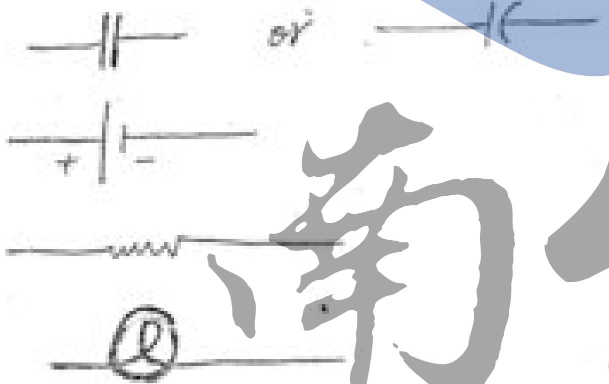
§ Symbols for Circuit Elements and Circuits

capacitor: or

battery:

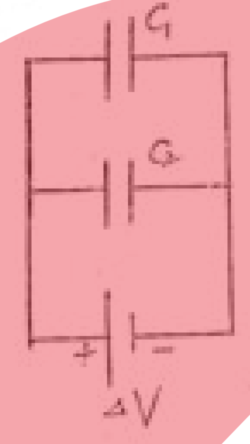
resistor:

lightbulb:



§16.7 Combinations of Capacitors

(a) Parallel Combination 並聯



$Q = Q_1 + Q_2$

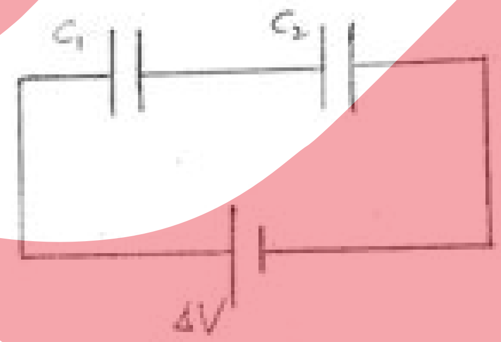
$Q = C_{eq} \Delta V$

$Q_1 = C_1 \Delta V$, $Q_2 = C_2 \Delta V$

$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$

$$\boxed{C_{eq} = C_1 + C_2}$$

(b) Series Combination 串聯



$\Delta V = \frac{Q}{C_{eq}}$

$\Delta V = \Delta V_1 + \Delta V_2$

$\Delta V_1 = \frac{Q}{C_1}$, $\Delta V_2 = \frac{Q}{C_2}$

$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Example 16.6

Example 16.7

Example 16.8

§16.9 Energy Stored in a Charged Capacitor

1. 電容器所增加的電位能 = V-q 關係曲線下的面積 (Q)

2. 電容器所增加的電位能 = $W = \int_0^Q \frac{q}{C}\,dq$ $\left(\because dW = \Delta V dq,\ \Delta V = \frac{q}{C}\right)$

電位能 $= \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

Example 16.9

§16.10 Capacitors with Dielectrics

定義：介電常數 (dielectric constant) K

$$= \frac{\text{加入電介質時的電容 } C}{\text{真空時的電容 } C_0}$$

$C_0 = \frac{\epsilon_0 A}{d}$, $C = KC_0 = \frac{K\epsilon_0 A}{d}$

註：ϵ_0 (真空中電容率) $= 8.85\times10^{-12}\ (C^2/N\cdot m^2)$

ϵ (介質中電容率) $= K\epsilon_0$

一般介質的 K值 >1 (注意：K值沒有單位) (另一定義：$K = \frac{\epsilon}{\epsilon_0}$)

Example 16.10