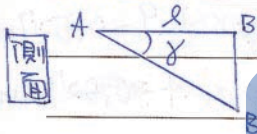


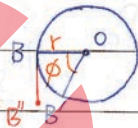
由幾何：



$$\overline{BB'} = l \tan \gamma$$

$$= l \gamma$$

端面



$$\widehat{BB'} = r \phi = \overline{BB''}$$

$$\Rightarrow \text{可得 } \overline{BB'} = \overline{BB''} = \widehat{BB'}$$

$$\Rightarrow l \gamma = r \phi \Rightarrow \gamma = \frac{r \phi}{l}$$

在軸中取另一半徑之圓柱分析可得 $\gamma = \frac{\phi}{2}$

$\Rightarrow$ 剪應變和軸斷面半徑之關係式

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在線彈性區內: Hookies law, 對於等向性材料: $\tau = G\gamma$

$\tau = G \frac{\phi}{L} \rho \Rightarrow \therefore$ 公式不含 T , 使用上極不方便

$\Rightarrow$ 希望得到 $\tau = ? T$

$$\tau = G \frac{\phi}{L} \rho = C \rho$$

為常數 $= C$

$\Rightarrow$ 由於在端面之分布可知分

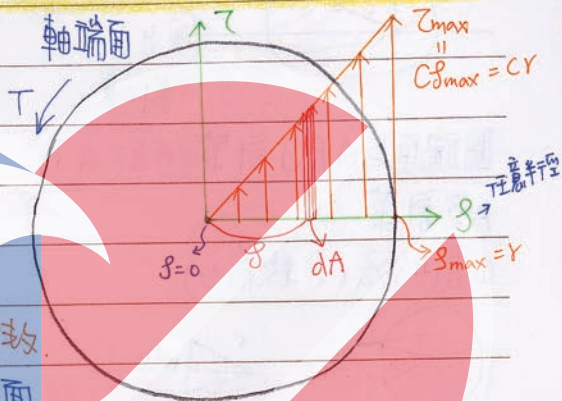
布為變數

$\Rightarrow \tau \propto \frac{\text{力}}{\text{面積}}$

$\Rightarrow$ 即整個面積之應力不是常數

$\Rightarrow$ 在端面半徑 ρ 處取一微小面

積 dA



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$\therefore$ 應力為均勻分布 $\Rightarrow \tau = \frac{\text{力}}{\text{面積}} = \frac{dV}{dA}$, 又 dV 對 O 點會產生一力矩

$\Rightarrow dT = \rho dV$

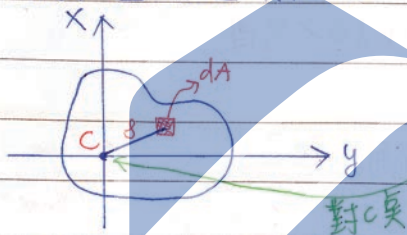
$$T = \int dT = \int \rho dV = \int \rho \tau dA = \int \rho C \rho dA$$

$$= C \int \rho^2 dA = CJ$$

$$\Rightarrow T = CJ = G \frac{\phi}{L} J \Rightarrow \phi = \frac{TL}{GJ}$$

$$\tau = C \rho = G \frac{\phi}{L} \rho = G \frac{\phi}{L} \frac{TL}{GJ} = \frac{T \rho}{J}$$

定義：面積極慣性矩

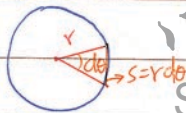


$$J_C = \int r^2 dA$$

圓端面之A及J計算

A之計算

① 扇形法 (變數為 θ)



$$A = \int dA = \int_0^{2\pi} \frac{1}{2} r^2 d\theta = \pi r^2$$

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$dA = \frac{1}{2} r^2 d\theta$
底 高