

7.3 Lengths of plane curves

1. Length of a function curve

Theorem: If f is continuously differentiable on $[a, b]$, the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$\int_a^b \sqrt{1 + (f'(x))^2} dx$$

Ex 1: Find the length of the curve $f(x) = x^{3/2}$ from $x = 0$ to $x = 4$.

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2. Length of a parametric curve

Theorem: If a curve $C: \begin{cases} x = x(t) \\ y = y(t) \end{cases}, t \in [a, b]$. $x'(t)$ and $y'(t)$ are continuous on $[a, b]$, then the length of C is

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt.$$

Ex 2: Find the length of the asteroid

$$x = \cos^3 t, \quad y = \sin^3 t, \quad t \in [0, 2\pi]$$



3. Length of a polar curve

Theorem: If the curve $r = f(\theta), \theta \in [\alpha, \beta]$ has a continuous first derivative, then the length of the curve is

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + (r'(\theta))^2} d\theta.$$

Ex 3: Find the length of the curve $r = e^{\theta}, \theta \in [0, \pi]$.

The logo of Southern Taiwan University, featuring a stylized, interlocking design in blue and red. The design consists of two large, curved shapes that overlap and form a central white space, resembling a stylized 'S' or a traditional Chinese motif.

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