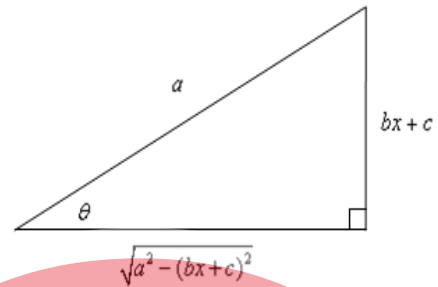


6.5 Trigonometric substitutions

Case 1 $\int R(x, \sqrt{a^2 - (bx + c)^2}) dx$ Let $bx + c = a \sin \theta$,

$$\sqrt{a^2 - (bx + c)^2} = \sqrt{a^2 - (a \sin \theta)^2} = a \cos \theta$$

$$dx = \frac{a}{b} \cos \theta d\theta.$$

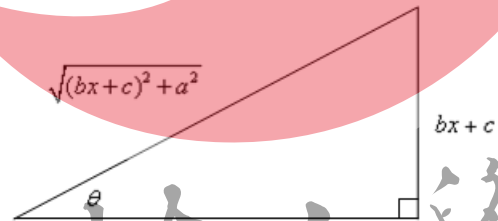


Ex1: $\int \frac{x^2}{\sqrt{9-x^2}} dx$

Case 2 $\int R(x, \sqrt{(bx + c)^2 + a^2}) dx$ Let $bx + c = a \tan \theta$,

$$\sqrt{(bx + c)^2 + a^2} = \sqrt{(a \tan \theta)^2 + a^2} = a \sec \theta$$

$$dx = \frac{a}{b} \sec^2 \theta d\theta.$$



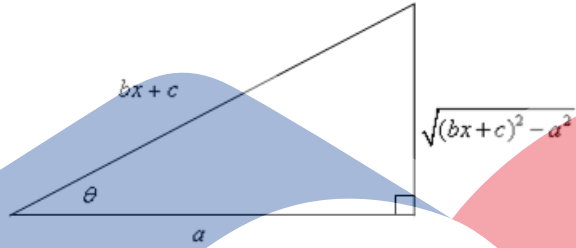
Ex2: $\int \frac{x^3}{\sqrt{1+x^2}} dx$

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Case 3 $\int R(x, \sqrt{(bx+c)^2 - a^2}) dx$

Let $bx+c = a \sec \theta$,

$$\sqrt{(bx+c)^2 - a^2} = \sqrt{(a \sec \theta)^2 - a^2} = a \tan \theta, \quad dx = \frac{a}{b} \sec \theta \tan \theta d\theta$$



Ex3: $\int \frac{1}{\sqrt{x^2 + 2x - 1}} dx$

Ex4: $\int_0^2 \frac{1}{(x^2 + 4)^2} dx$

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