

6.4 Trigonometric integrals

1. $\int \sin^m x \cos^n x dx$

Case 1 If m or n is odd.

Ex1: $\int \sin^3 x \cos^2 x dx$

Case 2 If Both m and n are even. Using

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x), \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Ex2: $\int_0^{\pi/2} \sin^2 x dx$

Ex3: $\int_0^{\pi/4} \sqrt{1 + \cos 4x} dx$

2. $\int \sec^m x \tan^n x dx, \int \csc^m x \cot^n x dx$

Case 1 If m is even.

Ex4: $\int \sec^6 x \tan x dx$

南台科技大學
Southern Taiwan University

Case 2 If n is odd and $m \geq 1$.

Ex5: $\int \sec^6 x \tan x dx$

Case 3 If $m = 0$.

Ex6: $\int \tan^3 x dx$

Case 4 If m is odd and n is even.

Ex7: $\int \sec^3 x dx$

3. $\int \sin ax \sin bxdx, \int \sin ax \cos bxdx, \int \cos ax \cos bxdx$

$$2 \sin mx \cos nx = \sin(m+n)x + \sin(m-n)x \quad (1)$$

$$2 \cos mx \cos nx = \cos(m+n)x + \cos(m-n)x \quad (2)$$

$$-2 \sin mx \sin nx = \cos(m+n)x - \cos(m-n)x \quad (3)$$

Ex8: $\int_{-\pi}^{\pi} \sin mx \sin nxdx, m \text{ and } n \in \mathbb{N}$