10.3 Partial Derivatives

Define: If $z = f(x, y)$, then

(1) the first partial derivative of f with respect to x is

$$\frac{\partial f}{\partial x}(x, y) \equiv f_x(x, y) \equiv \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

[regard y as a constant, differentiate with respect to x]

(2) the first partial derivative of f with respect to y is

$$\frac{\partial f}{\partial y}(x, y) \equiv f_y(x, y) \equiv \lim_{k \to 0} \frac{f(x, y+k) - f(x, y)}{k}$$

[regard x as a constant, differentiate with respect to y]

Ex 1: If $f(x, y) = x^2 y^3 - 2x^4 + y^5 - 3$, find f_x and f_y

Ex 2: If $g(u, v) = (u + 2v + 1)^3$, find $\frac{\partial g}{\partial u}(1, 0)$ and $\frac{\partial g}{\partial v}(1, 0)$

Ex 3: If $w = x^{y^z}$, find w_x, w_y, w_z

Define: Higher-Order Partial Derivatives

(1) $f_{xx}(x, y) = \frac{\partial}{\partial x} f_x(x, y) = \lim_{h \to 0} \frac{f_x(x+h, y) - f_x(x, y)}{h}$

(2) $f_{xy}(x, y) = \frac{\partial}{\partial y} f_x(x, y) = \lim_{k \to 0} \frac{f_x(x, y+k) - f_x(x, y)}{k}$

(3) $f_{yx}(x, y) = \frac{\partial}{\partial x} f_y(x, y) = \lim_{h \to 0} \frac{f_y(x+h, y) - f_y(x, y)}{h}$

(4) $f_{yy}(x, y) = \frac{\partial}{\partial y} f_y(x, y) = \lim_{k \to 0} \frac{f_y(x, y+k) - f_y(x, y)}{k}$

Note: (1) $f_x = \dfrac{\partial f}{\partial x}$, $f_y = \dfrac{\partial f}{\partial y}$

(2) $f_{xx} = \dfrac{\partial^2 f}{\partial x^2}$, $f_{xy} = \dfrac{\partial^2 f}{\partial y \partial x}$, $f_{yx} = \dfrac{\partial^2 f}{\partial x \partial y}$, $f_{yy} = \dfrac{\partial^2 f}{\partial y^2}$

Ex 4: If $f(x,y) = xe^y + x^3y^2 + 3x$, find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$

Theorem: If f_{xy} and f_{yx} are continuous on an open disc R, then

$$f_{xy}(a,b) = f_{yx}(a,b), \quad \forall (a,b) \in R.$$

Ex 5: If $f(x,y) = \sin(x+2y)$, find f_{xyx}

Ex 6: If $w = \ln\sqrt{x^2+y^2+z^2}$, find $w_{xx} + w_{yy} + w_{zz}$

Ex 7: If $f(x,y,z) = ye^x + x^3 \ln z - e^z \sin y$, find f_{xxz}, f_{xyz}