

10.2 Limits and continuity

Define: Let $f(x, y)$ be defined except possibly at (x_0, y_0) on an open disc centered at (x_0, y_0) , then

$$\lim_{(x,y) \rightarrow (x_0, y_0)} f(x, y) = L \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, \exists \text{ if } 0 < |(x, y) - (x_0, y_0)| < \delta \\ \Rightarrow |f(x, y) - L| < \varepsilon$$

Ex 1: $\lim_{(x,y) \rightarrow (1,2)} \frac{3xy^2}{x^2 + y^2}$

Ex 2: $\lim_{(x,y) \rightarrow (0,0)} \frac{3xy^2}{x^2 + y^2}$

Concept: If $x = r \cos \theta, y = r \sin \theta$, then $(x, y) \rightarrow (0, 0) \Leftrightarrow r \rightarrow 0$.

Ex 3: $\lim_{(x,y) \rightarrow (0,0)} \frac{1}{x^2 + y^2}$

Ex 4: $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

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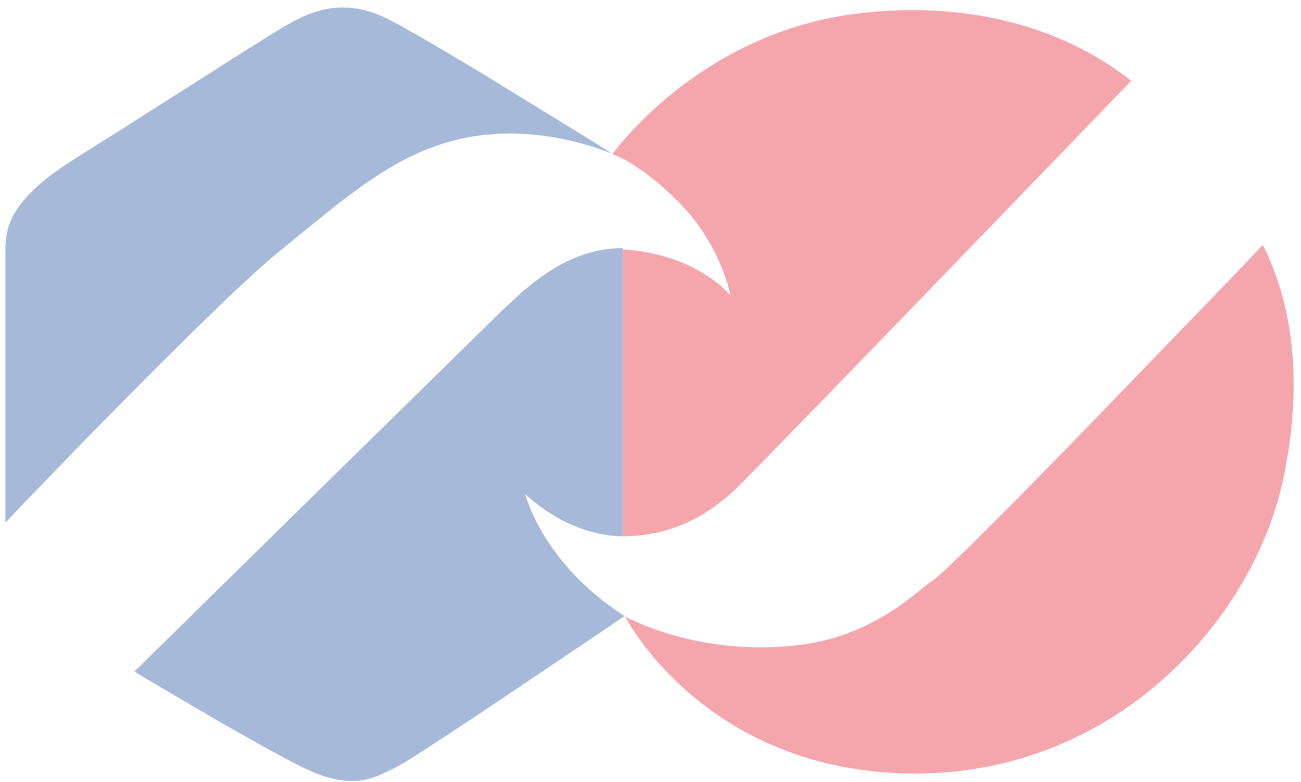
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Define: If $f(x, y)$ is continuous at (x_0, y_0)

$$\Leftrightarrow \lim_{(x,y) \rightarrow (x_0, y_0)} f(x, y) = f(x_0, y_0)$$

$$\Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, \exists \text{ if } |(x, y) - (x_0, y_0)| < \delta \\ \Rightarrow |f(x, y) - f(x_0, y_0)| < \varepsilon$$

Ex 5: Discuss the continuity of $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$.



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