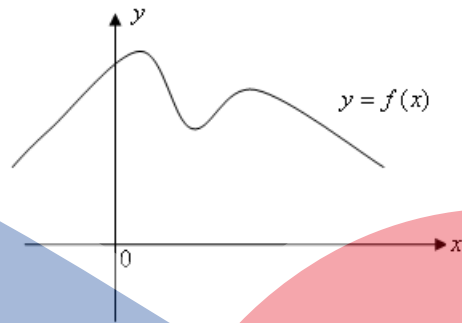


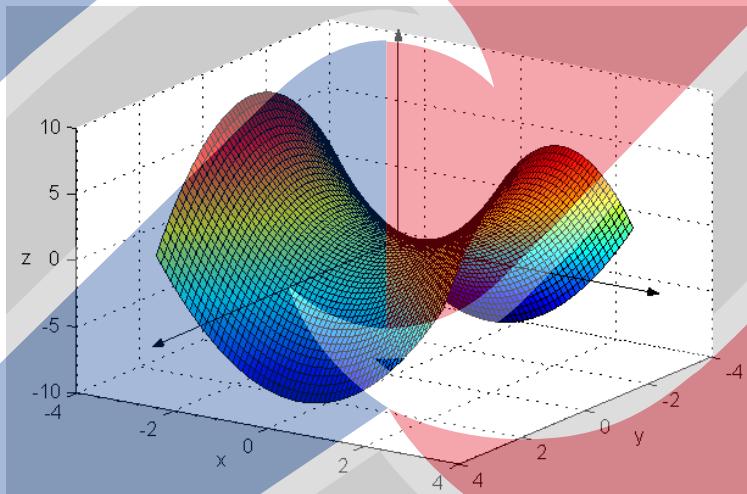
10 Partial derivatives

10.1 Functions of several variables

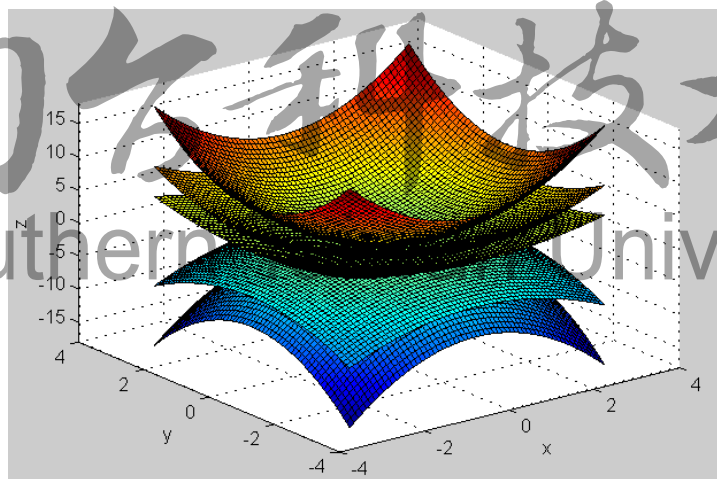
Define: $y = f(x) \rightarrow y$ is a function of simple variable x . (The graph is a curve)



$z = f(x, y) \rightarrow z$ is a function of two variables x, y . (The graph is a surface)



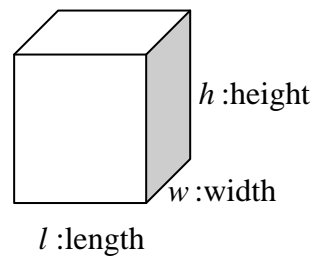
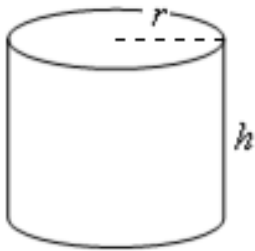
$w = f(x, y, z) \rightarrow w$ is a function of three variables. (The graph is a manifold)



$u = f(x_1, x_2, \dots, x_n) \rightarrow u$ is a function of n variables.

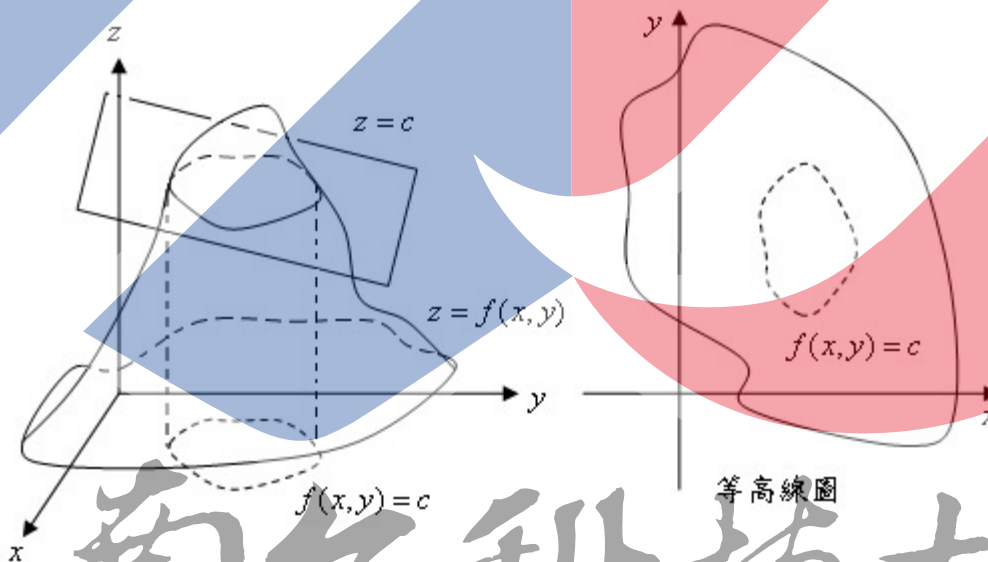
Ex 1: The volume of a right circular cylinder is $V(r, h) = \pi r^2 h$.

The volume of a rectangular solid is $V(l, w, h) = lwh$.



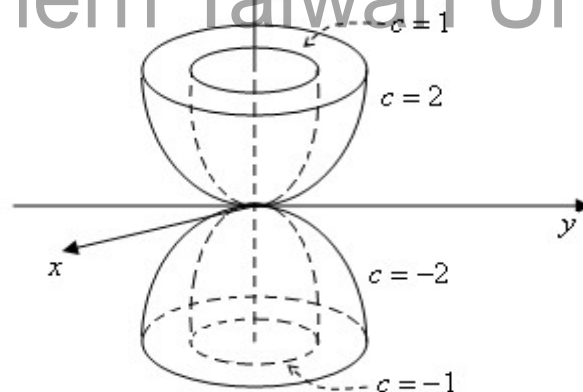
Ex 2: If $f(x, y) = \frac{\sqrt{3-x^2-y^2}}{x}$, find $f(1, -1)$ and its the domain.

Def: (1) For $z = f(x, y)$ the graph of the equation $z = c$ ($f(x, y) = c$) is called a level curve (等高線) of the function f .

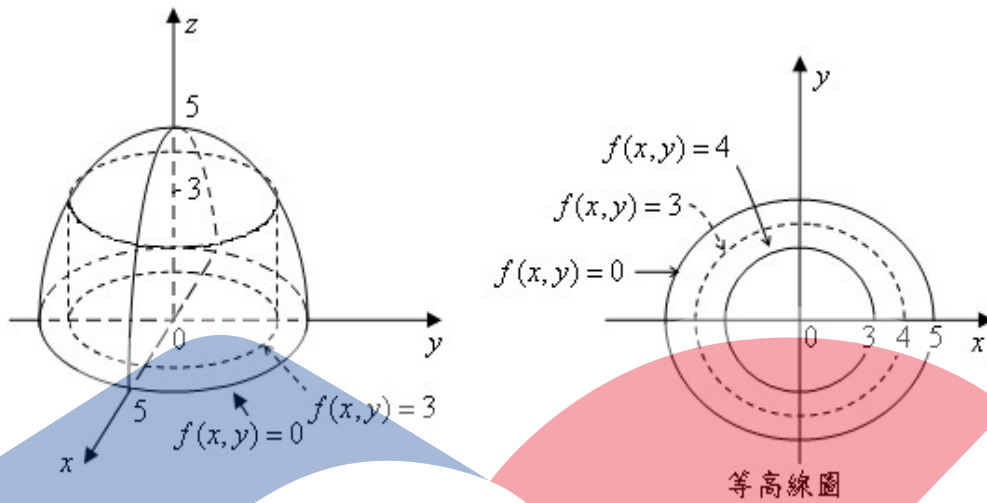


(2) For $w = f(x, y, z)$ the graph of the equation $w = c$ is called a level surface (等高面) of the function f .

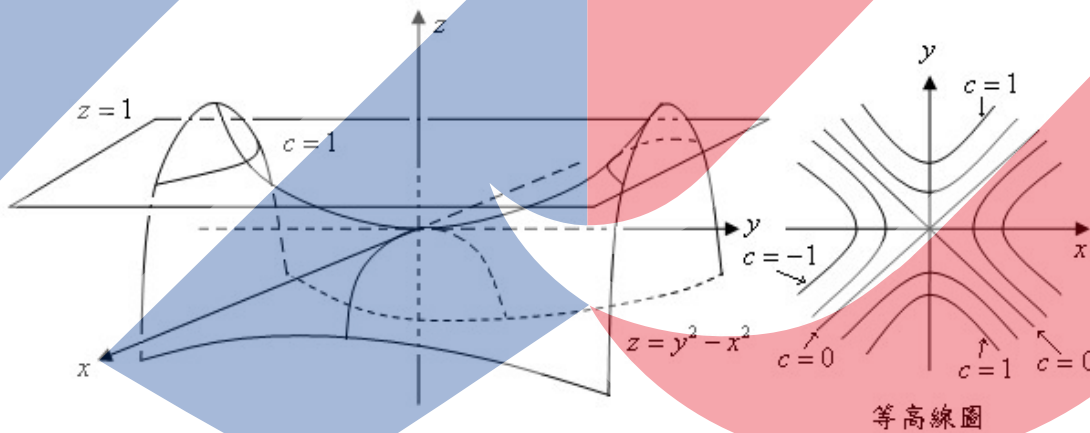
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Ex 3: Sketch the graph of $f(x, y) = \sqrt{25 - x^2 - y^2}$.



Ex 4: Sketch the graph of $z = y^2 - x^2$. (Hyperbolic paraboloid, 雙曲拋物面)



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