

4.7 Differentials

Def: If $y = f(x)$, dx is called the differential of x . Then the differential of y is

$$dy = f'(x)dx.$$

Concept: Derivative of $f(x)$ exists \Leftrightarrow Differential of $f(x)$ exists.

$$[\because df(x) = f'(x)dx]$$

Differential formulas:

Let u and v be differentiable functions of x ,

$$\Rightarrow (1) d(cu) = cdu, \forall c \in \mathbb{R}.$$

$$(2) d(u \pm v) = du \pm dv$$

$$(3) d(uv) = u dv + v du$$

$$(4) d\frac{v}{u} = \frac{u dv - v du}{u^2}, u \neq 0.$$

Ex 1: $y = (x^2 + x)(x^4 - 5x)$. Find dy

Ex 2: If $x = \sqrt{u+1}$, find du

Using differentials to approximate function values:

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$$

(1) A point of view by mathematic:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

(2) A point of view by geometric:

$$\text{Let } \Delta x = dx$$

$$\Rightarrow \Delta y \approx dy$$

$$\therefore f(x_0 + \Delta x) - f(x_0) \approx f'(x_0)\Delta x$$

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Def: (1) Actual change (實際變量) in y : $\Delta y = f(x_0 + \Delta x) - f(x_0)$

(2) Approximate change (近似變量) in y : $dy = f'(x_0)\Delta x$

(3) $L(x) = f(x_0) + f'(x_0)(x - x_0)$ is the linearization of f at x_0 .

Ex 3: Use differentials to approximate $\sqrt[5]{33}$.

Ex 4: Let $f(x) = x^2 + 1$.

- (1) Find the differential of f .
- (2) Find the approximate change in y if x changes from 1 to 1.02
- (3) Find the actual change in y if x changes from 1 to 1.02

Ex 5: Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.

The logo of Southern Taiwan University is a stylized, interlocking design. It consists of two main shapes: a blue one on the left and a red one on the right, both with white curved cutouts. The shapes are intertwined, creating a sense of movement and connection.

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