4.7 Differentials

Def: If $y=f(x)$, dx is called the differential of x. Then the differential of y is

$$dy=f'(x)dx.$$

Concept: Derivative of $f(x)$ exists $\Leftrightarrow$ Differential of $f(x)$ exists.

$$[\because df(x)=f'(x)dx]$$

Differential formulas:

Let u and v be differentiable functions of x,

$\Rightarrow$ (1) $d(cu)=cdu$, $\forall c\in\mathbb{R}$.

(2) $d(u\pm v)=du\pm dv$

(3) $d(uv)=udv+vdu$

(4) $d\dfrac{v}{u}=\dfrac{udv-vdu}{u^2}, u\neq 0$.

Ex 1: $y=(x^2+x)(x^4-5x)$. Find dy

Ex 2: If $x=\sqrt{u+1}$, find du

Using differentials to approximate function values:

$$f(x_0+\triangle x)\approx f(x_0)+f'(x_0)\triangle x$$

(1) A point of view by mathematic:

$$f'(x_0)=\lim_{\triangle x\to 0}\frac{f(x_0+\triangle x)-f(x_0)}{\triangle x}\approx\frac{f(x_0+\triangle x)-f(x_0)}{\triangle x}$$

(2) A point of view by geometric:

Let $\triangle x=dx$

$\Rightarrow \triangle y\approx dy$

$\therefore\ f(x_0+\triangle x)-f(x_0)\approx f'(x_0)\triangle x$

Def: (1) Actual change (實際變量) in y: $\triangle y=f(x_0+\triangle x)-f(x_0)$

(2) Approximate change (近似變量) in y: $dy=f'(x_0)\triangle x$

(3) $L(x)=f(x_0)+f'(x_0)(x-x_0)$ is the linearization of f at x_0.

Ex 3: Use differentials to approximate $\sqrt[5]{33}$.

Ex 4: Let $f(x) = x^2 + 1$.

(1) Find the differential of f.

(2) Find the approximate change in y if x changes from 1 to 1.02

(3) Find the actual change in y if x changes from 1 to 1.02

Ex 5: Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$.