4.3 Applications of second derivatives

Def: Concavity(凹性)

(1) If $f'(x):\nearrow$ on $(a,b) \Rightarrow$ f is concave upward(上凹) on (a,b). ($f:\cup$ on (a,b))
(2) If $f'(x):\searrow$ on $(a,b) \Rightarrow$ f is concave downward(下凹) on (a,b). ($f:\cap$ on (a,b))

Theorem: Test for concavity
(1) If $f''(x) > 0$ $\forall x \in (a,b) \Rightarrow$ $f:\cup$ on (a,b).
(2) If $f''(x) < 0$ $\forall x \in (a,b) \Rightarrow$ $f:\cap$ on (a,b).

Def: If concavity of f changes at $x = c$, and f is continuous at $c \Rightarrow (c, f(c))$ is an inflection point (反曲點).

Theorem: (Points of inflection)
If $(c, f(c))$ is an inflection point of $f \Rightarrow$ $f''(c) = 0$ or does not exist.

Ex 1: $f(x) = x^3$, $f(x) = x^{\frac{3}{5}}$.

Concept: The converse proposition is wrong.
Ex 2: $f(x) = x^4$.

Ex 3: Determine the points of inflection and discuss the concavity of $f(x) = x^4 - 4x^3$.

Theorem: (The second derivative test)

Let $f'(c) = 0$ and $f''(x)$ be continuous near c.

(1) If $f''(c) > 0 \Rightarrow f(c)$ is a local minimum.

(2) If $f''(c) < 0 \Rightarrow f(c)$ is a local maximum.

Concept:

Note: This test fails when $f'(c)$ does not exist or $f'(c) = f''(c) = 0$.

Ex 4: Find the local extrema for $f(x) = x^4 - 4x^3$.