4.2 The first derivative test

Def: Increasing and decreasing functions

If $\forall\ x_1, x_2 \in [a,b]$ and $x_1 < x_2$, we have

(1) $f(x_1) \le f(x_2) \, (f(x_1) < f(x_2))$ ➔ f is increasing (strict increasing) on $[a,b]$.

(f :↗ on $[a,b]$)

(2) $f(x_1) \ge f(x_2) \, (f(x_1) > f(x_2))$ ➔ f is decreasing (strict increasing) on $[a,b]$.

(f :↘ on $[a,b]$)

Theorem: Increasing/decreasing test

(1) If $f'(x) > 0 \ \ \forall x \in (a,b) \implies f$ is increasing on (a,b).

(2) If $f'(x) < 0 \ \ \forall x \in (a,b) \implies f$ is decreasing on (a,b).

(3) If $f'(x) = 0 \ \ \forall x \in (a,b) \implies f$ is constant on (a,b).

Ex 1: Find the intervals on which $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing or decreasing.

Corollary: If $f'(x) = g'(x), \forall x \in \mathbb{R}$, $\Rightarrow \exists c \in \mathbb{R}, \ni f(x) = g(x) + c, \forall x \in \mathbb{R}$.

Theorem: (The first derivative test)

Let c be a critical point of f, if $\exists\ \delta > 0, \ni$

(1) $f'(x) > 0,\ x \in (c-\delta, c)$ and $f'(x) < 0,\ \forall x \in (c, c+\delta)$ ➔ $f(c)$ is a local maximum.

(2) $f'(x) < 0,\ x \in (c-\delta, c)$ and $f'(x) > 0,\ \forall x \in (c, c+\delta)$ ➔ $f(c)$ is a local minimum.

Ex 2: Find the relative extrema of $f(x) = (x^2 - 4)^{2/3}$.

Ex 3: Find the relative extrema of $f(x) = \frac{1}{2}x - \sin x$ on $(0, 2\pi)$.