3.2 Differentiation Formulas

Theorem:

(1) $\frac{d}{dx}c = 0, \quad \forall c \in \mathbb{R}$

$f(x) = c, \; m = f'(x) = 0$

Ex 1: a. $(3)' = 0$ b. $\frac{d}{dx}e^{\pi} = 0$

(2) $\frac{d}{dx}x^n = nx^{n-1}, \quad \forall n \in \mathbb{N}$

(3) $\frac{d}{dx}x^r = rx^{r-1}, \quad \forall r \in \mathbb{R}$

Ex 2: a. $(x)' = 1$, b. $(x^8)' = 8x^7$, c. $(x^{\frac{5}{2}})'$, d. $(\sqrt{x})'$, e. $(\frac{1}{\sqrt[3]{x}})'$

Ex 3: $\frac{d}{dx}\frac{\sqrt{x}}{\sqrt[5]{x^3}}$

Theorem: If f and g are both differentiable, then

(1) $(cf(x))' = cf'(x), \quad \forall c \in \mathbb{R}$

Ex 4: a. $(5x^3)'$, b. $(\frac{3}{\sqrt{x}})'$

(2) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

Ex 5: a. $f(x) = 4x^3 - 7x^2 + 8x - 2$, find $f'(x)$

b. $g(t) = \frac{t^2}{5} + \frac{5}{t^3}$, find $g'(1)$

Ex 6: Show that the equation of the line $y = mx + b$ has the slope m.

Ex 7: $\frac{d}{dx}\frac{x^5-3x+4\sqrt{x}}{x}$

(3) $(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Ex 8: If $f(x) = x^3(\sqrt{x}+1)$, find $f'(x)$

Ex 9: If $f(x) = (2x^2-1)(x^3+5x)$, find $f'(x)$

Extended form: $(f\cdot g\cdot h)' = f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h'$

Ex 10: If $y = (x^2+1)(x^2+2)(x^2+3)$, find $y'(1)$

(4) $(\frac{f(x)}{g(x)})' = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g^2(x)}$, $g(x)\neq 0$

Ex 11: Find $f'(x)$ if $f(x) = \frac{x}{2x-3}$

Ex 12: If $f(x) = \frac{x^2+1}{x^2-1}$, find $f'(x)$

Extended form: $(\frac{1}{g})' = \frac{-g'}{g^2}$, $g\neq 0$

Ex 13: $\frac{d}{dx}\frac{1}{x^2-1}$

Ex 14: If $f(x)=\begin{cases} x^2, x\geq 1 \\ ax+b, x<1 \end{cases}$ and f is differentiable at $x=1$, find a,b

Ex 15: Find the slope and an equation of the tangent line to the graph of $f(x)=2x+\frac{1}{\sqrt{x}}$ at the point $(1,3)$.

Ex 16: If $f(x)=x^8-7x+3$, find $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$