2.3 Limits involving infinity

1. infinite limits(無窮極限)

Ex 1: Find (1) $\lim_{x\to 0^+}\frac{1}{x}$ (2) $\lim_{x\to 0^-}\frac{1}{x}$ (3) $\lim_{x\to 0}\frac{1}{x}$

Ex 2: Find $\lim_{x\to 1} e^{\frac{x-2}{x-1}}$

2. limits at infinite (在無窮遠的極限)

Concept: (1) $\lim_{x\to\infty}\frac{1}{x}=0$ $\left[\!\left[\frac{1}{\infty}=0\right]\!\right]$

(2) $x << x^2 << x^3$ as $x\to\infty$

Ex 3: (1) $\lim_{x\to\infty}\frac{3x^2+5x-2}{x^3-2x^2-6}$ (2) $\lim_{x\to\infty}\frac{x^3-2x^2-6}{3x^2+5x-2}$ (3) $\lim_{x\to\infty}\frac{x^3-2x^2-6}{3x^3+5x-2}$

Ex 4: $\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{x+3}$

Ex 5: $\lim_{x\to\infty}(\sqrt{x^2-x}-\sqrt{x^2+x})$

Concept: (1) $\infty+\infty=\infty, k+\infty=\infty, k-\infty=-\infty, \infty-\infty$ (indeterminate form)

(2) $\infty\bullet\infty=\infty, \infty\bullet(-\infty)=-\infty, k\bullet\infty=\begin{cases}\infty, k>0\\ -\infty, k<0\end{cases}, 0\bullet\infty$ (indeterminate form)

(3) $\frac{k}{\infty}=0, \frac{1}{\infty}=0, \frac{\infty}{\infty}$ (indeterminate form)

$\frac{1}{0^+}=\infty, \frac{1}{0^-}=-\infty, \frac{0}{0}$ (indeterminate form)

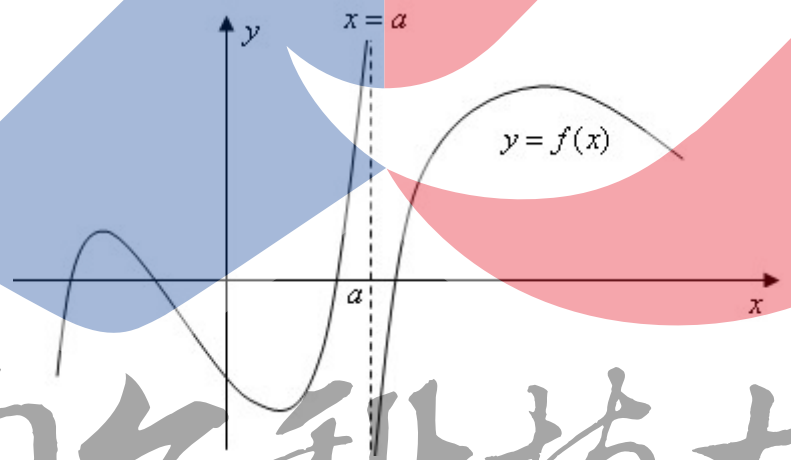
(4) $\infty^k=\begin{cases}\infty, k>0\\ 0, k<0\end{cases}, \infty^0$ (indeterminate form)

$1^k=1, 1^\infty$ (indeterminate form), $k^0=1, k\neq 0$, 0^0 (indeterminate form)

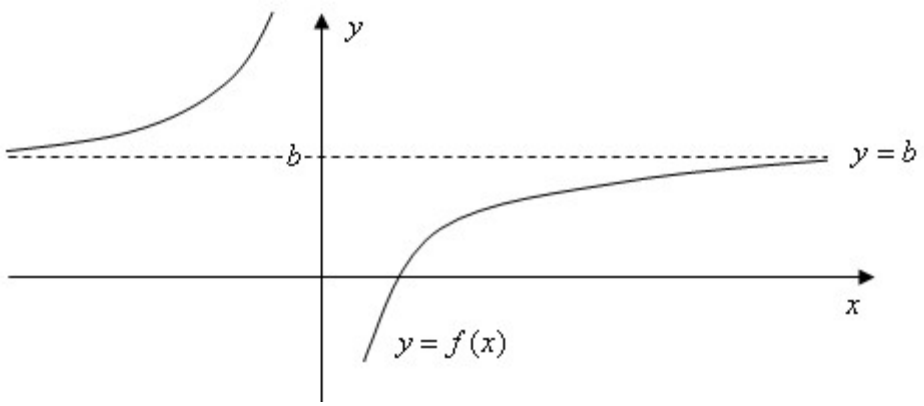
Ex 6: $\lim_{x\to 1^+}\frac{1}{1+2^{\frac{1}{1-x}}}$

3. Asymptotes (漸近線)

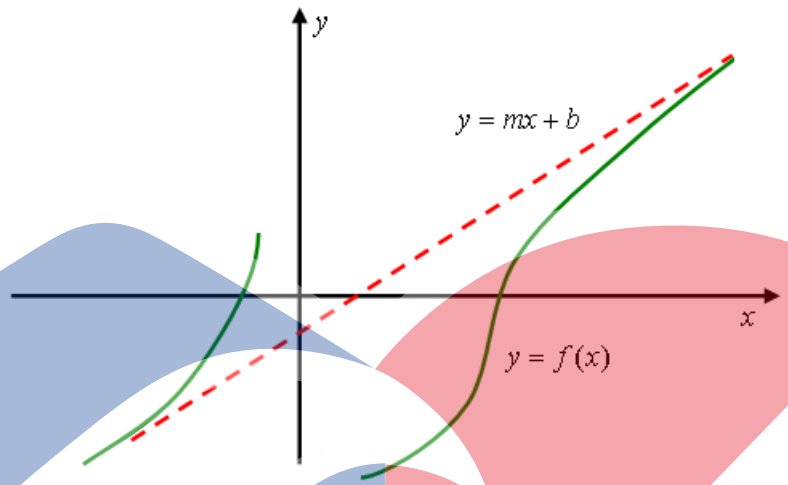
Def: (1) If $\lim_{x\to\infty(-\infty)} f(x)=b \Rightarrow y=b$ is a horizontal asymptote (H.A.,水平漸近線) of f.



(2) If $\lim_{x\to a^+(a^-)} f(x)=+\infty(-\infty) \Rightarrow x=a$ is a vertical asymptote (V.A.,垂直漸近線) of f.



(3) If $\lim_{x\to\infty(-\infty)}[f(x)-(mx+b)]=0 \Rightarrow y=mx+b$ is a slant asymptote (S.A.,斜漸近線) of f.



How to find $y=mx+b$?

Method 1: If $f(x)=mx+b+R(x)$ and $\lim_{x\to\infty(-\infty)}R(x)=0$.

Method 2: $m=\lim_{x\to\infty(-\infty)}\frac{f(x)}{x}$ and $b=\lim_{x\to\infty(-\infty)}[f(x)-mx]$.

Concept: If $f(x)=\frac{q(x)}{p(x)}$ is a rational function.

(1) $\deg p(x)\geq\deg q(x) \Rightarrow f$ has horizontal asymptotes.

(2) $p(a)=0\wedge q(a)\neq 0 \Rightarrow$ the line $x=a$ is a vertical asymptote.

(3) $\deg p(x)+1=\deg q(x) \Rightarrow f$ has slant asymptotes.

Ex 7: Find all asymptotes for $f(x)=\frac{x+1}{x-2}$.

Ex 8: Find all asymptotes for $f(x)=\frac{2x^2+3x}{x-1}$.