2.2 One-sided limits

Def: (1) The left-hand limit: the limit of $f(x)$ as x approaches a from values less that a is equal to L, we denote

$$\lim_{x\to a^-} f(x)=L \quad \text{or} \quad f(x)\to L \ \text{ as } \ x\to a^-.$$

(2) The left-hand limit: the limit of $f(x)$ as x approaches a from values greater that a is equal to R, we denote

$$\lim_{x\to a^+} f(x)=R \quad \text{or} \quad f(x)\to R \ \text{ as } \ x\to a^+.$$

Theorem: $\lim_{x\to a} f(x)=L \Leftrightarrow \lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=L$

If $\lim_{x\to a^+} f(x)\neq \lim_{x\to a^-} f(x) \Rightarrow \lim_{x\to a} f(x)$ does not exist.

Ex 1: The Heaviside function is defined by

$$H(t)=\begin{cases}0, & t<0\\ 1, & t\geq 0\end{cases}$$

Find (1) $\lim_{t\to 0} H(t)$ (2) $\lim_{t\to 1} H(t-1)$ (3) $\lim_{t\to 0} t^2 H(t)$

Ex 2: If $f(x)=\begin{cases}\sqrt{x}, & x>0\\ 1, & x=0\\ -x, & x<0\end{cases}$, find $\lim_{x\to 0} f(x)$

Ex 3: Find $\lim_{x\to 1}\dfrac{x-1}{|x-1|}$

Ex 4: Find $\lim_{x\to n}[x] \quad \forall n\in\mathbb{Z}$