1.5 Trigonometric functions and inverse trigonometric functions

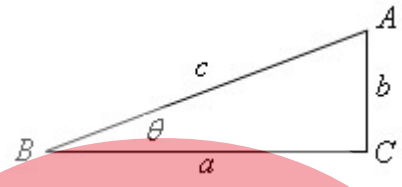
1. Trigonometric functions

Def: If $0 < \theta < \frac{\pi}{2}$, then

$$\sin\theta = \frac{b}{c},\ \cos\theta = \frac{a}{c}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{b}{a},\ \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{a}{b}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{c}{a},\ \csc\theta = \frac{1}{\sin\theta} = \frac{c}{b}$$

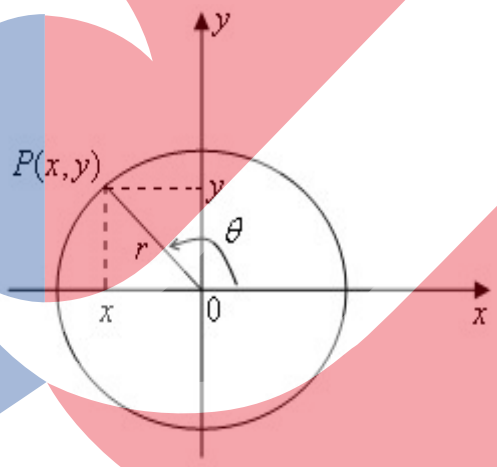


Def: Let $r = \sqrt{x^2 + y^2}$

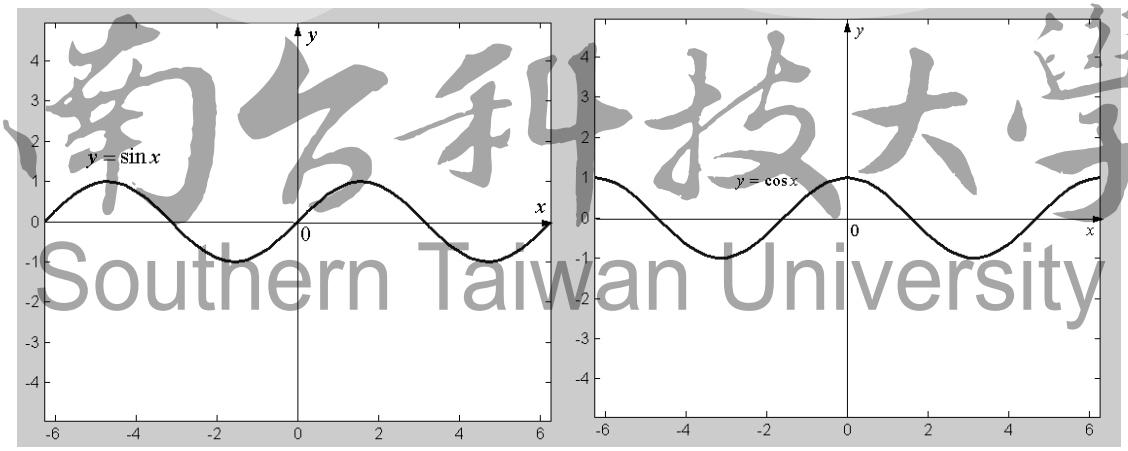
$$\sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r}$$

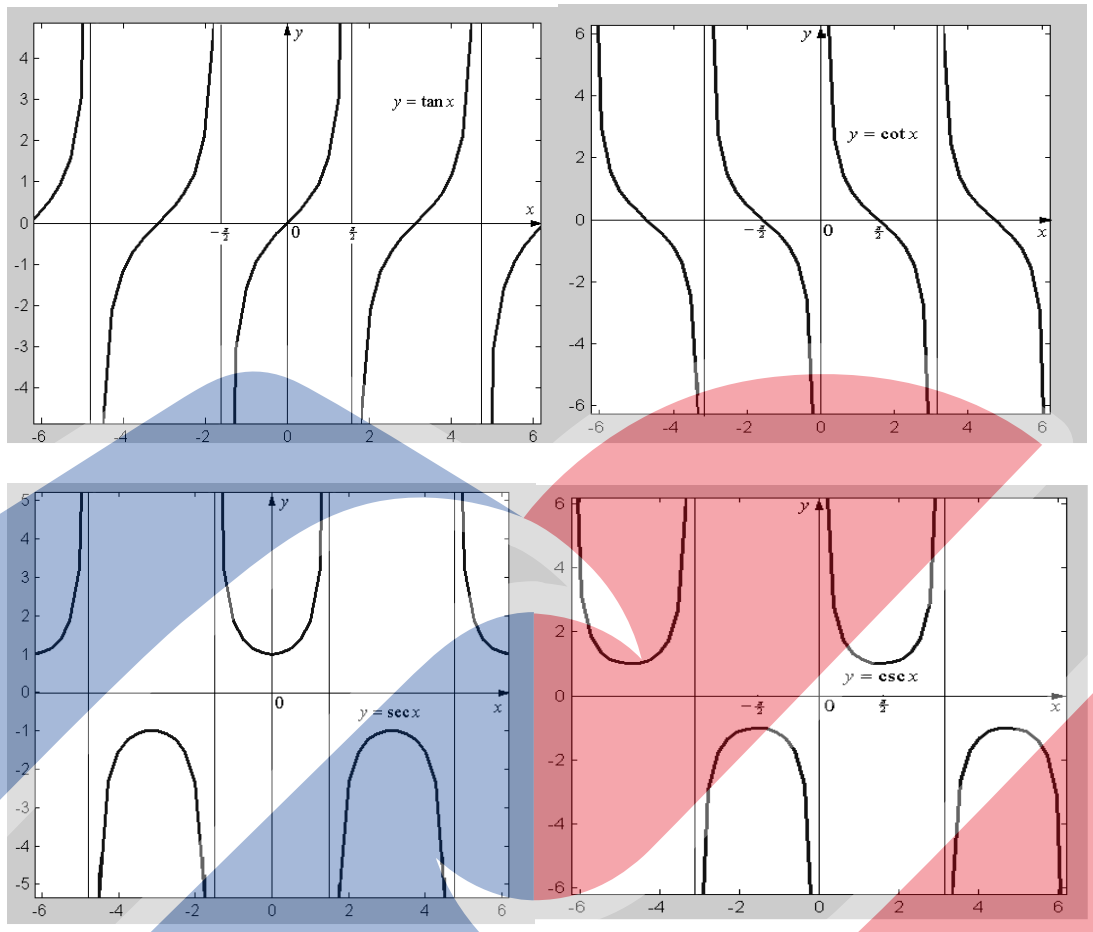
$$\tan\theta = \frac{y}{x},\ \cot\theta = \frac{x}{y}$$

$$\sec\theta = \frac{r}{x},\ \csc\theta = \frac{r}{y}$$

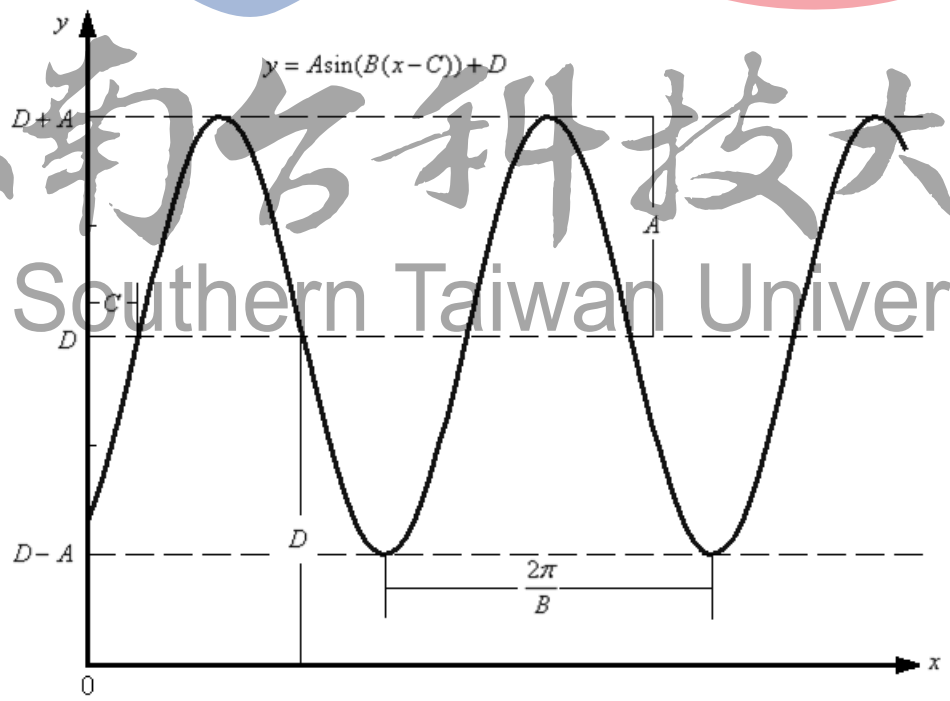


Graphs:





Def: If $y = A\sin(B(x-C))+D$, $|A|$ is the amplitude(振幅), $\frac{2\pi}{|B|}$ is the period(週期), C is the horizontal shift(水平平移), D is the vertical shift(垂直平移).

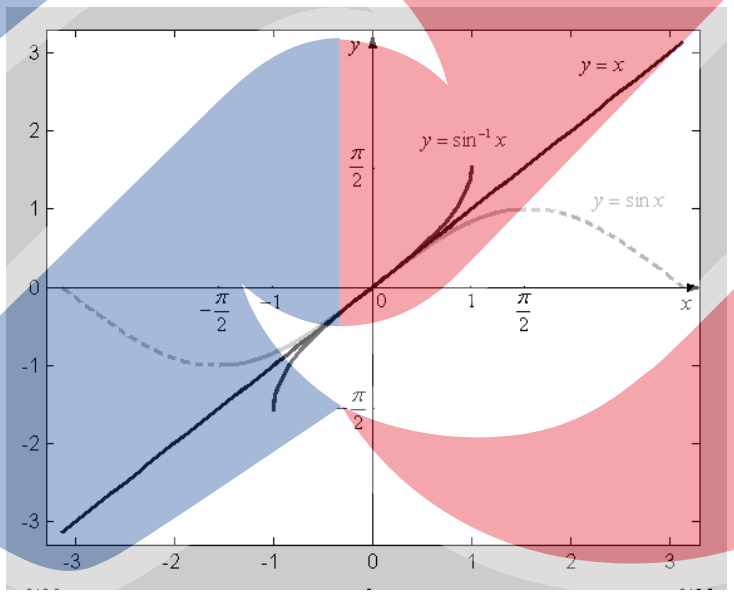


Ex 1: Sketch the graph of $y = 3\sin(2x)$

2. Inverse trigonometric functions

Def: $y = \sin^{-1} x \equiv \arcsin x \Leftrightarrow x = \sin y$

$$\sin^{-1} : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$$

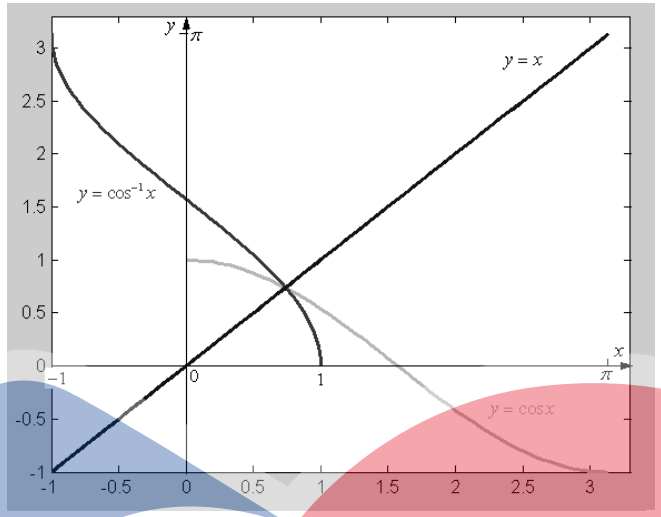


concept: $\sin^{-1}(-x) = -\sin^{-1} x$

Ex 2: Find (1) $\sin^{-1}\frac{1}{2}$ (2) $\sin^{-1}(-\frac{1}{2})$ (3) $\tan(\sin^{-1}\frac{1}{3})$

Def: $y = \cos^{-1} x \Leftrightarrow x = \cos y$

$$\cos^{-1} : [-1,1] \to [0, \pi]$$

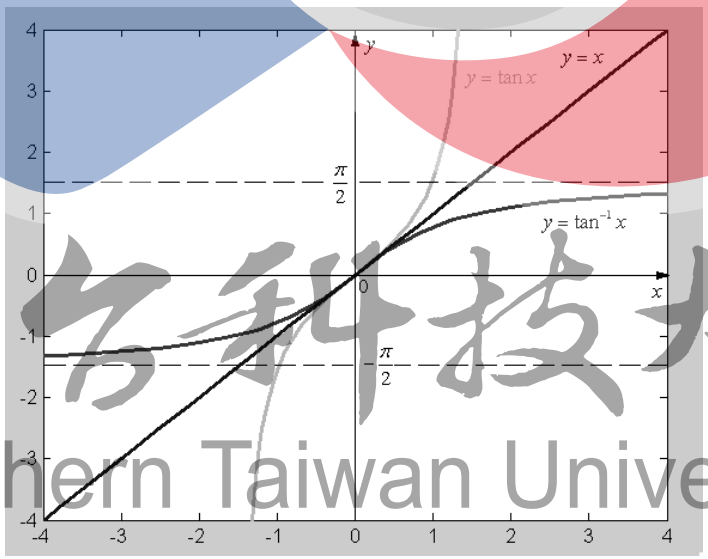


concept: $\cos^{-1}(-x) = \pi - \cos^{-1} x$

Ex 3: Find $\cos^{-1}(-\frac{1}{2})$

Def: $y = \tan^{-1} x \Leftrightarrow x = \tan y$

$\tan^{-1} : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$



concept: (1) $\tan^{-1}(-x) = -\tan^{-1} x$

(2) $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$, $\lim_{x\to-\infty} \tan^{-1} x = -\frac{\pi}{2}$

Ex 4: Evaluate $\cos(\tan^{-1} x)$

Def: $y = \cot^{-1} x \Leftrightarrow x = \cot y$ and $\cot^{-1} : \mathbb{R} \to (0, \pi)$

$y = \sec^{-1} x \Leftrightarrow x = \sec y$ and $\sec^{-1} : (-\infty, -1] \cup [1, \infty) \to [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$y = \csc^{-1} x \Leftrightarrow x = \csc y$ and $\csc^{-1} : (-\infty, -1] \cup [1, \infty) \to [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$