1.3 Inverse function

Def: $f(x)$ is 1-1 function $\Leftrightarrow$ $\forall\ x_1, x_2 \in D_f$ *and* $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$\Leftrightarrow$ $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Ex 1: $f(x) = x^2, x \in \mathbb{R}$ is not 1-1 function.

Def: inverse function(反函數)

If $f : A \to B$ is 1-1 $\Rightarrow$ $\exists\ g : f(A) \subset B \to A$, ∋

$g(f(x)) = x, \quad \forall\ x \in A$

$f(g(y)) = y, \quad \forall\ y \in f(A)$.

Where g is called the inverse function of f and denoted by $g = f^{-1}$.

Concept:

(1) $y = f(x) \Leftrightarrow x = f^{-1}(y)$

$\therefore\ f^{-1}(f(x)) = x, \quad \forall\ x \in D_f$

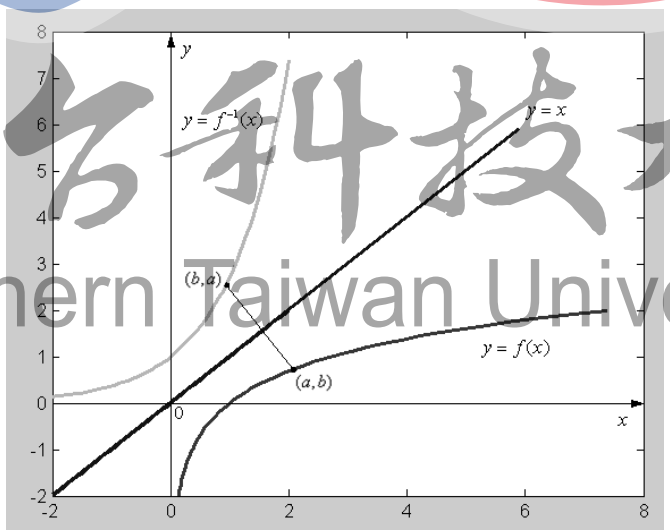
$f(f^{-1}(x)) = x, \quad \forall\ x \in R_f$

(2) The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are symmetric about the line $y = x$.

$[\![\because$ *If* $(a,b) \in \{(x,y) \mid$ *the graph of* $y = f(x)\}$

$\Leftrightarrow (a,b) \in \{(x,y) \mid$ *the graph of* $x = f^{-1}(y)\}$

$\Leftrightarrow (b,a) \in \{(x,y) \mid$ *the graph of* $y = f^{-1}(x)\}$ $]\!]$



Ex 2: If $f(x) = 2x^3 - 1$, find the inverse of f.