Ordinary Differential Equations

Chapter1 Classification of Differential Equations and Their Solutions

I. Differential equations

A differential equation is an equation which involves differentials or derivatives.

Ex1 $y' = \cos x,$ (1.1)

$y'' + 4y = 0,\ xdx + ydy = 0,$ (1.2)

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \quad (1.3)$$

II. Classification of differential equations

1. Ordinary differential equations: Differential equations, in which there is a single independent variable, are known as ordinary differential equations. Equations (1.1) and (1.2) are ordinary differential equations.

2. Partial differential equations: If an equation involves more than one independent variable, so that partial derivatives enter, it is known as a partial differential equation. Equation (1.3) is a partial differential equation.

3. Linear differential equation: A linear differential equation is that there is no products or nonlinear functions of dependent variable and its derivatives.

4. Nonlinear differential equation: A differential equation containing products or nonlinear functions of dependent variable and its derivatives is a nonlinear differential equation.

5. Quasi-linear differential equation: A nonlinear differential equation is quasi-linear when it is linear in its highest derivative.

Ex2 $x^2\frac{d^2y}{dx^2}+2y=2$ is linear. (1.4)

$\frac{dy}{dx}=x+y^2$ is nonlinear. (1.5)

$\frac{d^2y}{dx^2}+\sin y=0$ is nonlinear. (1.6)

$\frac{d^2y}{dx^2}+y^2=0$ is quasi-linear. (1.7)

III. The order and degree of differential equations

1. Order: The order of a differential equation is the order of the highest derivative.

2. Degree: The degree of a differential equation is the degree of the highest derivative entering, when the equation has been rationalized and cleared of fractions.

Ex3 $y'+y\tan x=\sin 2x$ is first order and first degree. (1.8)

$(y-xy')^2=k^2(1+y'^2)$ is first order and second degree. (1.9)

$y''^2=(1+y'^2)^3$ is second order and second degree. (1.10)

IV. Solutions

A relation, connecting the independent and dependent variables, which satisfies the differential equation identically, is called a solution of a differential equation.

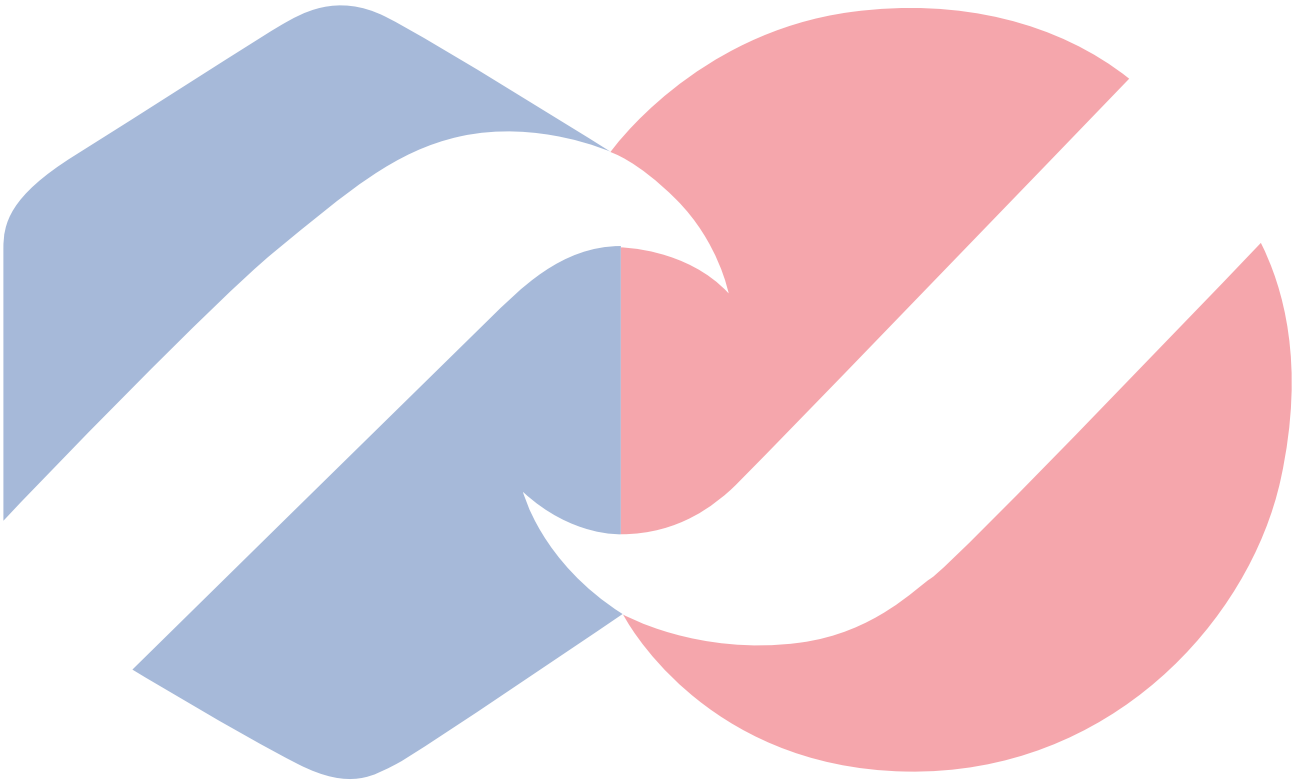
1. General solution: A solution involving the maximum number of arbitrary constants is called the general solution.

2. Particular solution: A solution which is derivable from the general solution by assigning fixed values to the arbitrary constants is called a particular solution.

3. Singular solution: A solution which cannot be obtained from the general solution is called a singular solution. A singular solution can occur only in the solution of nonlinear differential equations.

V. The problems to be studied in our course

1. Differential equations of the first order and first degree.
2. Differential equations of the first order and higher degree.
3. Differential equations of higher order.
4. Series solutions of differential equations.



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