7.4 Areas of surfaces of revolution

1. Function form

Theorem: If $f(x) \ge 0, x \in [a,b]$ is continuously differentiable, the area of the surface generated by revolving the curve $y = f(x)$ about the x-axis is

$$S = 2\pi \int_0^L y ds = 2\pi \int_a^b y\sqrt{1+(y')^2}dx = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}dx$$

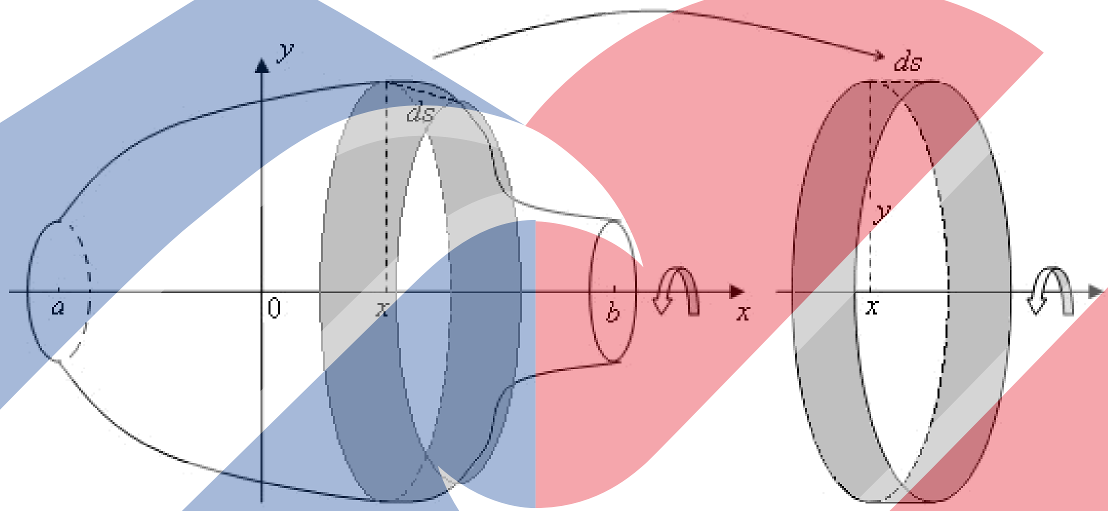

Figure 7.19

Theorem: If $f(x), x \in [a,b], a \ge 0$ is continuously differentiable, the area of the surface generated by revolving the curve $y = f(x)$ about the y-axis is

$$S = 2\pi \int_0^L x ds = 2\pi \int_a^b x\sqrt{1+(y')^2}dx = 2\pi \int_a^b x\sqrt{1+(f'(x))^2}dx$$

Ex 1: The circle $x^2 + y^2 = r^2$ is rotated about the x-axis to generate a sphere. Find its surface area.

Ex 2: Find the area of the surface generated by revolving the curve $y = x^2, x \in [1,2]$,

about the y-axis.

2. Parametric form

Theorem: If the curve $C: \begin{cases} x = x(t) \\ y = y(t) \end{cases}, t \in [a,b]$. $x'(t)$ and $y'(t)$ are continuous on $[a,b]$, then the areas of the surfaces generated by revolving the curve C about

(1) the x-axis ($y \ge 0$):

$$S = 2\pi \int_0^L y ds = 2\pi \int_a^b y \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

(2) the y-axis ($x \ge 0$):

$$S = 2\pi \int_0^L x ds = 2\pi \int_a^b x \sqrt{(x'(t))^2 + (y'(t))^2} dt.$$

Ex 3: Find the area of the surface generated by revolving the curve $x = \cos t, y = 1 + \sin t, 0 \le t \le 2\pi$, about the x-axis.