

3.7 Higher-Order Derivatives

$$\text{First derivative : } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = D_x f(x) = \frac{d}{dx} f(x)$$

$$\text{Second derivative : } f''(x) = (f')'(x) = \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h} = D_x^{(2)} f(x) = \frac{d^2}{dx^2} f(x)$$

$$\text{Third derivative : } f'''(x) = (f'')'(x) = \lim_{h \rightarrow 0} \frac{f''(x+h) - f''(x)}{h} = D_x^{(3)} f(x) = \frac{d^3}{dx^3} f(x)$$

$$\text{Fourth derivative : } f^{(4)}(x) = (f''')'(x) = \lim_{h \rightarrow 0} \frac{f'''(x+h) - f'''(x)}{h} = D_x^{(4)} f(x) = \frac{d^4}{dx^4} f(x)$$

$$\vdots$$

$$\text{nth derivative : } f^{(n)}(x) = (f^{(n-1)})'(x) = \lim_{h \rightarrow 0} \frac{f^{(n-1)}(x+h) - f^{(n-1)}(x)}{h} = D_x^{(n)} f(x) = \frac{d^n}{dx^n} f(x)$$

Ex 1: If $y = x^4 + x^3 + x^2 + x + 1$, find $y^{(n)}$

Ex 2: If $y = x^k, k \in \mathbb{N}$. Find $y^{(n)}$

Ex 3: If $f(x) = \frac{1}{2x+3}$, find $f^{(n)}(x)$ and $f^{(2009)}(0)$

Ex 4: If $f(x) = \sum_{n=1}^{\infty} \frac{1}{n+1} x^n$, find $f^{(98)}(0)$

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Ex 5: Find y'' if $y = (2x^2 + 3)^{\frac{3}{2}}$.

Ex 6: If $x^2 + y^2 = 1$, find y''

Ex 7: If $f(x) = |x^2 - 4|$, find $f'(x), f''(x)$

The logo of Southern Taiwan University is a stylized, abstract design. It consists of two large, overlapping, curved shapes. The left shape is blue and the right shape is red. They are connected at their inner curves, forming a central white space. The overall shape resembles a stylized 'S' or a pair of interlocking curves.

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