3.3 The chain rule

Ex 1: If $y=(x^2+x+1)^3$, find $\dfrac{dy}{dx}$

Sol 1:

$$\frac{dy}{dx}=(x^2+x)'(x^2+x)(x^2+x)+(x^2+x)(x^2+x)'(x^2+x)+(x^2+x)(x^2+x)(x^2+x)'$$
$$=3(x^2+x)^2(x^2+x)'$$
$$=3(x^2+x)^2(2x+1)$$

Sol 2:

Let $u(x)=x^2+x,\Rightarrow y=u^3$

So, $\dfrac{dy}{du}=3u^2$ and $\dfrac{du}{dx}=2x+1$

Thus, $\dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}=3u^2\cdot(2x+1)=3(x^2+x)^2(2x+1)$

Theorem:(The chain rule)

If $y=f(u)$, and $u=g(x)$ are differentiable, then $y=f(g(x))$ is differentiable and

$$\left(f(g(x))\right)'=f'(g(x))\cdot g'(x)$$

or

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

Extended form:

If $y=f(g(h(x))), u=g(h(x)), v=h(x)$

➔ $(f(g(h(x))))'=f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$

or

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$$

Concept: If $y=(f(x))^r, r\in\mathbb{R}$, then $y'=r(f(x))^{r-1}\cdot f'(x)$

Ex 2: Let $F(x)=(3x+1)^2$. Compute $F'(x)$ and $F'(1)$.

Ex 3: Differentiate the function $G(x) = \sqrt{x^2 + 1}$

Ex 4: Find $f'(x)$ if $f(x) = \dfrac{1}{(4x^2 - 7)^2}$

Ex 5: If $f(x) = (2x^2 + 3)^4 (3x - 1)^5$, find $f'(x)$

Ex 6: If $y = u^{3/4} + u^2$, $u = x^4 - 3x^2$, find $\dfrac{dy}{dx}$

Ex 7: Find the slope of the tangent line to the graph of the function

$$f(x) = \left(\frac{2x+1}{3x+2} \right)^3$$

at the point $(0, \frac{1}{8})$.

Ex 8: If $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$, find y'