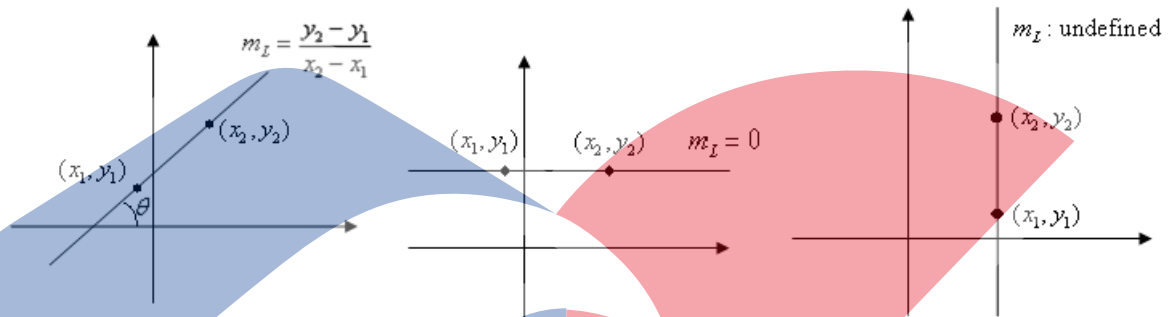## 1.2 Equations of lines

Def: Slope(斜率) of a line

If (x_1, y_1) and (x_2, y_2) are any two distinct points on a line L, then

(1) the slope of L is

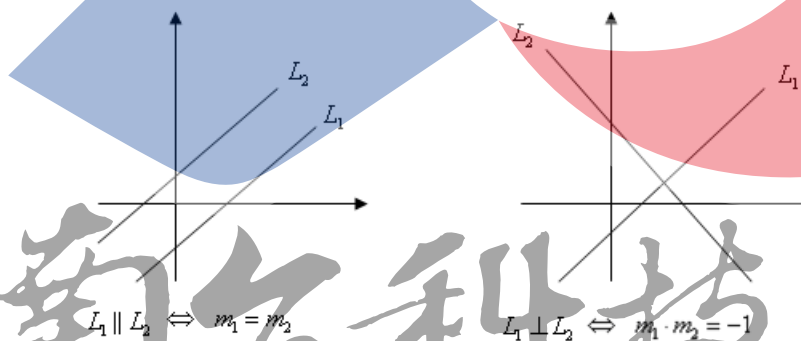$$m_L = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta, \text{ if } x_1 \neq x_2.$$

(2) the slope of L is undefined if $x_1 = x_2$.

Theorem: If m_1 and m_2 are the slopes of the lines L_1 and L_2, respectively, then

(1) L_1 is parallel(平行) to L_2 $(L_1 \parallel L_2) \Leftrightarrow m_1 = m_2$

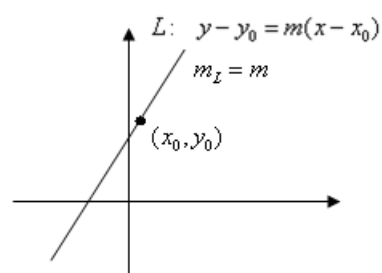(2) L_1 is perpendicular(垂直) to L_2 $(L_1 \perp L_2) \Leftrightarrow m_1 \cdot m_2 = -1$.

Equations of lines:

(1) Point-slope form(點斜式)

If L has slope m and passes through (x_0, y_0), then an equation of L is given by
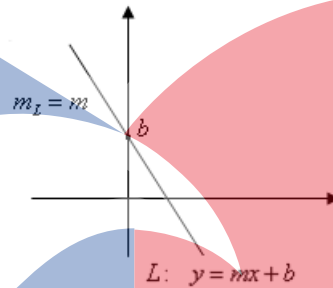
$$L: \quad y - y_0 = m(x - x_0)$$

Ex1: Find an equation of the line that passes through the point $(1,-2)$ and has slope 4.

(2) Slope-intercept form(斜截式)

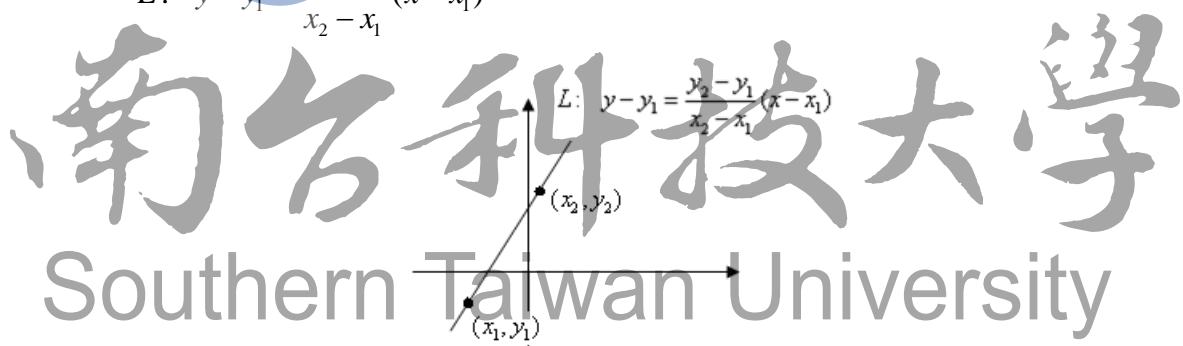 If L has slope m and y-intercept b, then

$$L: \quad y = mx + b$$

Ex2: Find an equation of the line that has slope 3 and y-intercept -4.

(3) Two-points form(兩點式)

  If L passes through the points (x_1, y_1) and (x_2, y_2), then

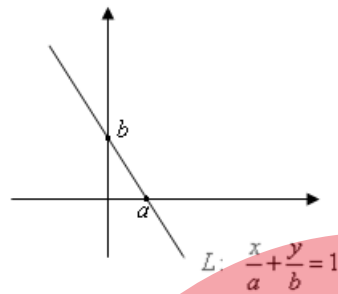$$L: \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Ex3: Find an equation of the line that passes through $(1,-2)$ and $(2,3)$.

(4) Intercept form(截距式)

If a and b are the $x-$ and $y-$intercepts, respectively, then

$$L:\ \frac{x}{a}+\frac{y}{b}=1,\ \ ab\neq 0.$$

$$L:\ \frac{x}{a}+\frac{y}{b}=1$$

Ex4: Find an equation of the line with the $x-$intercepts 3 and $y-$intercepts -5.

(5) General form(一般式)

$L:\ ax+by+c=0$ is an equation of a straight line.

Then $m_L=-\dfrac{a}{b},\ \ y-$intercepts $-\dfrac{c}{b}.$

Ex5: Find an equation of the line that passes through the point $(-2,3)$ and is perpendicular to the line with equation $2x+3y-1=0$.